



## The Park Place Economist

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Volume 22 | Issue 1

Article 10

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2014

# A Gravitational Model of Crime Flows in Normal, Illinois: 2004-2012

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### Recommended Citation

Bates, Jake K. '14 (2014) "A Gravitational Model of Crime Flows in Normal, Illinois: 2004-2012," *The Park Place Economist*: Vol. 22

Available at: <http://digitalcommons.iwu.edu/parkplace/vol22/iss1/10>

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# A Gravitational Model of Crime Flows in Normal, Illinois: 2004-2012

## **Abstract**

This study intends to use these past contributions as a framework for a gravity model of crime in the Town of Normal, Illinois from the years 2004 to 2012. Following Smith (1976), Elffers et al. (2008), and Walker (2009), data has been gathered from the local police department and includes a variety of violent crime and property crime. Following Elffers et al.'s (2008) analysis, geographical area and a measure of the residential housing stock will be used as control variables; similarly to the approach followed by Smith (1974), Elffers et al. (2008) and Kahane (2013), the distance between police sub-beats in Normal is approximated by estimating the geographical distance between centroids. Because each of the aforementioned analyses found distance to be a reliable predictor of the concentration of crime across towns and countries, this study expects to establish the same relationship.

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# A Gravitational Model of Crime Flows in Normal, Illinois: 2004-2012

Jake Bates

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## I. Introduction

Criminal activity has posed social and economic costs everywhere since the development of civil society and sociologists and psychologists are often inclined to study the physical and mental determinants of crime. Their findings can help rehabilitate criminals and may proactively prevent some crime, but law enforcement officers are nevertheless kept busy protecting the populations and properties they serve. In Normal, Illinois there were 13 more robberies reported in 2012 than in the previous year, but 21 fewer reported incidents of domestic violence. Yet, in 2010, according to USA.com, Normal recorded a crime index of 1,282.34—relatively low compared to its twin city, Bloomington (1,509.53), and the average across the state of Illinois (1,746.73). The study of widely varying patterns in criminal behavior is critical to assisting law enforcement efforts, and the field of economics has begun to develop methods that can add to social and psychological understandings of crime.

As Brantingham (2011) notes, evolving research theories and technologies have connected criminal activity and geographical location. Smith (1976) explained the predictive power that geographical distance has on the concentration of crime in Rochester, New York with a “gravity model” relating the two variables. This model has since been modified and applied elsewhere and offers

valuable insights into how to anticipate and prevent criminal activity. This paper will adapt Smith’s model and apply it to crime in the Town of Normal, Illinois from January 2004 to December 2012. A total of 17,759 crimes recorded across 16 police-patrolled sub-beats during this seven-year sample will be used to investigate seasonal and cyclical trends in crime as well as the flow, or mobility, of crime between specific areas of town.

The empirical relationship between crime and place has been explored more frequently in recent decades as technology has simplified geographical coding and computations. This increased interest has helped to explain patterns in criminal activity as it changes across time and location. As Brantingham (2011) states, this information should be taken seriously in policy-making and economic development because, in her words, “a city helps shape crime patterns and crime patterns help shape a city.” A gravity model methodology has proved useful to scholars across countries and across time in explaining the inverse relationship between concentration of crime in two areas and the geographical distance between the areas.

The classical gravity model was first applied to economic theory by Jan Tinbergen, who modified Isaac Newton’s “Law of Universal Gravitation,” first published in 1687. Newton calculated gravitational force as the product of a gravitational constant and two objects’ masses divided by the distance

between the objects squared, meaning that gravitational force is inversely related to distance. Tinbergen sought to apply a similar model to international trade, predicting trade flows as the product of a constant and two countries' gross domestic products divided by the distance between them raised to a  $\theta$  power, to be estimated statistically. Walker and Unger (2009) recall that at the time Tinbergen had no specific theory to defend modifying the gravity model this way, but that it came to be preferred because its predictive power was much greater than any theory or model that came before it. In this way, empirical analysis guided economic theory and the gravity model was subsequently used to predict flow of migrants and tourists, foreign direct investment, and eventually patterns of crime.

In the first application of a gravity model to crime flows, Smith (1976) notes that crime is subject to "inverse distance variations" and so a gravity model is apt for correlating the density of crime across town and the distances between locations of crime. Smith (1976) applies his theory to the city of Rochester, New York in 1972 by extracting a list of all crimes that ended in an arrest recorded by the Rochester Police Department. Then, coding the locations of each criminal's residence and the location of the crime by census block and estimating the geographical center of each census block, he calculates an estimate of the distance between the two locations. After testing several models, Smith concludes the classical gravity model has the most predictive power in estimating crime and that "crime...is subject to the general class of inverse distance variations formulated as gravity laws."

Other studies tend to follow Smith's methodology closely. Elffers et al. (2008) conducts a similarly scaled study of The Hague, The Netherlands, incorporating an intervening opportunities theory. Using eight years of data including over 60,000 crimes

recorded by the local police department, measuring distance between a criminal's residence and the location of the crime as the distance between the centers of each respective neighborhood, and controlling for the land area and populations of the neighborhoods, they also find statistically significant results and explain over sixty percent of the variance in crime.

Walker and Unger (2009) and Kahane (2013) carry out studies using gravity models to predict one aspect of crime on a more macroeconomic scale. Walker and Unger (2009) sought to predict the proceeds of money laundering crimes using data from Australia in 1996. Their findings indicate that their "Walker gravity model" and its "attractiveness and distance indicator" are reliable and valid in predicting the flow of laundered money. Kahane (2013) analyzes the flow of guns used to commit crimes across the United States. He uses data collected by the Bureau of Alcohol, Tobacco, Firearms, and Explosives in 2009 which includes records of over 40,000 guns used to commit crimes in states other than the state where they are first sold. He approximates distance between states using their geographical centers and finds that, including measures of state gun laws and gang activity, a gravity model is able to explain part of the flow of guns between states.

This study intends to use these past contributions as a framework for a gravity model of crime in the Town of Normal, Illinois from the years 2004 to 2012. Following Smith (1976), Elffers et al. (2008), and Walker (2009), data has been gathered from the local police department and includes a variety of violent crime and property crime. Following Elffers et al.'s (2008) analysis, geographical area and a measure of the residential housing stock will be used as control variables; similarly to the approach followed by Smith (1974), Elffers et al. (2008) and Kahane (2013), the distance between police sub-beats

in Normal is approximated by estimating the geographical distance between centroids. Because each of the aforementioned analyses found distance to be a reliable predictor of the concentration of crime across towns and countries, this study expects to establish the same relationship.

The rest of the paper is organized as follows: Section II will describe the data and methodology used in the analysis. Section III will detail the results of the regression, and Section IV will discuss the implications of our findings, including extensions for future research.

## II. Data and Methods

To test the research hypothesis that concentration of crime between sub-beats in Normal, Illinois is inversely related to the distance between the sub-beats, this paper will use a panel data set including sixteen sub-beats, six types of crime, and 108 months—or nine full years—from January 2004 to December 2012. The Normal Police Department makes available online the number of thefts, batteries, vehicle and residential burglaries, robberies, and sexual assaults—all stratified by the month and location of occurrence. Though not an exhaustive list of crimes, they are among the most common in the Town of Normal and include a variety of property crimes and violent crimes. Each crime poses different physical and financial costs: sexual assaults, batteries, and robberies all include the use or threat of force and vehicle and residential burglaries require unauthorized entry into property. A total of 17,763 crimes were recorded and used in this analysis; Figure 1 plots the sum of these crimes series by month throughout the seven-year sample. The average number of crimes per month was 165.42, with a maximum of 245 recorded in October 2006 and August 2010 and a minimum of 77 recorded in February 2011. Figure 2 separates the data by type of crime into six series. Theft

were the most commonly reported crime, totaling 7,339. During the same time, there were 5,525 batteries, 2,302 vehicle burglaries, 2,040 residential burglaries, 297 robberies, and 260 sexual assaults. As expected, these crimes were not evenly distributed across the town. Figure 3 reports the average number of total crimes per year recorded in each of 16 police sub-beats. The area most affected by crime, sub-beat 33, recorded over 260 crimes per year during this sample while sub-beat 30 to its west recorded less than eight crimes per year.

Monthly means were calculated in order to test for seasonal patterns in frequency of crime and they are reported in Figure 4. As expected, following empirically established patterns in crime, winter months generally have fewer crimes reported than the summer and autumn months. Over this seven year sample, February recorded fewer crimes than any other month, averaging 121 total crimes. October had a larger concentration of crime than any other month, 199.2 crimes on average, which is likely informed by the high frequencies of crime in summer months continued through the beginning of Illinois State University's school year, which adds approximately 20,000 potential criminals and victims of crime to the Town of Normal's population.

A 12-month moving average was used to determine whether or not cycles in the frequency of crime could be observed during our sample. The results are shown in Figure 5 where two cycles are evident. The first expansion in crime began in early 2006 and lasted until mid-2008. This was followed by a contraction in crime rates until the end of 2009. A second year-long expansion lasted until late 2010 and was followed by a contraction lasting until the end of the sample in 2012.

In order to test the effects of distance, this study follows methods similar to Smith



(1976) and Elffers (2008) who recorded distances between geographical centers of census blocks and neighborhoods. Centroids of the 16 patrolled police sub-beats in Normal were estimated after outlining their geographical boundaries and the distance between each pair of sub-beats was estimated as the distance between the two geographical centers.

### III. Results

Bearing in mind that seasonal and cyclical effects are evident in crime rates, it is necessary to induce linearity and stationarity in the data set before conducting a linear regression. An Augmented Dickey-Fuller (ADF) test was used to test for the presence of a unit root in the series in levels. The results are reported in Table 1 and indicate that we can reject with greater than 99 percent confidence the hypothesis that there is a unit root in the series. Next, the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test for stationarity was used for the series in levels and indicates that with greater than 95 percent confidence we can conclude the series is not stationary, or random in variance, around a trend. The results are shown in Table 2. Thus, in order to induce stationarity and to test and report growth rates in crime across different sub-beats, the logarithmic values of the series were calculated and subsequently used to compute first-order differences. Thus, each variable is measured as the percentage change in crime between months. These growth rates in crime are plotted in Figure 6. The ADF and KPSS tests of first-order differences in logarithmic levels are reported in Tables 3 and 4 and indicate that the series has no unit roots and is stationary, making it fit for an unbiased Ordinary Least Squares linear regression analysis.

Because this study seeks to analyze the impact of distance between sub-beats on the density of crime, it is necessary to manipulate these series further in order to

control for intervening factors likely to affect crime density. Given the data available from the Normal Police Department and the lack of advanced Geographical Information Systems (GIS) capabilities, it has not been possible to calculate the same “gravity term” that other scholars have employed—see Smith (1976). Instead, the residential area of each sub-beat was approximated in square miles and the logarithmic first-order difference series in each sub-beat was divided by the residential area. This transformation normalizes the changes in crime by both the size of each sub-beat and an approximate measure of how densely populated they are. As a result, because sub-beat 30 has no residential area and its series would therefore be divided by zero, it has been dropped from further analysis. Notice that sub-beat 30 had the least reported crime across this time sample and is located on the far west side of town, so this omission is unlikely to detract from analysis of the rest of the data set.

Regression equations were calculated for each of the 15 sub-beats with residential areas larger than zero. The dependent variable in each equation is the percentage change in total crime between months in any given sub-beat, divided by the residential area. The fourteen independent variables are the percentage change in crime in each other sub-beat, divided by the residential area, and then divided by the distance between the dependent and independent sub-beats. This last transformation of the independent variables accounts for the ease of mobility of crime between sub-beats. That is, a short distance between the dependent and independent variable will artificially augment the change in crime in the independent sub-beat relative to an independent sub-beat that is further away. This transformation, standard in the literature, accounts for the fact that sub-beats far away from a dependent sub-beat should have less influence on the density of crime in the dependent sub-beat than those that

are adjacent to it. Then, each of the fifteen regression equations is written as:

$$\left(\frac{\% \Delta \text{Crime}_A}{\text{Area}_A}\right) = C + \frac{\left(\frac{\% \Delta \text{Crime}_B}{\text{Area}_B}\right)}{D_{AB}} + \frac{\left(\frac{\% \Delta \text{Crime}_C}{\text{Area}_C}\right)}{D_{AC}} + \frac{\left(\frac{\% \Delta \text{Crime}_D}{\text{Area}_D}\right)}{D_{AD}} + \dots + \frac{\left(\frac{\% \Delta \text{Crime}_O}{\text{Area}_O}\right)}{D_{AO}} + \varepsilon$$

Crime rates in thirteen out of fifteen sub-beats were found to have a statistically significant relationship with crime in at least one other sub-beat. Among these 13 sub-beats, there were a total of 31 statistically significant relationships in crime rates. As expected, most of these relationships were positive, meaning crime rates moved together in most areas. However, seven negative relationships may inform us about movement of criminal activity between areas. The sign of these relationships, and the geographical location of the individual sub-beats, illustrates the mobility and flow of crime in the Town of Normal during this sample. The equations with the most explanatory power and largest numbers of statistically significant relationships are summarized in Tables 5, 7, and 9 with corresponding maps illustrating the results in Figures 7, 8, and 9 and residual diagnostics reported in Tables 6, 8, and 10.

Changing crime rates in sub-beat 41, located in the Town's southeast, were substantially correlated with changing crime rates in six other sub-beats. Sub-beat 21, to the north, had the only statistically significant negative relationship; a coefficient of -0.58 indicates that with a 10 percent decrease in crime in sub-beat 21, crime rates in sub-beat 41 increase by 5.8 percent. This negative relationship suggests that as crime in sub-beat 21 decreases (possibly as a result of increased police patrolling), offenders might concentrate into sub-beat 41. The other statistically significant relationships, in sub-beats 10, 11, 12, 13, and 32 were all positive, meaning that crime rates generally move in the same direction in these sub-beats and in sub-beat 41. Crime rates in sub-beat 12, to the northwest,

report the largest estimated coefficient, and thus exert the largest influence

on crime rates in sub-beat 41. Specifically, a 10 percent increase in crime in sub-beat 12 leads to a 22.89 percent increase in crime in sub-beat 41. In sum, changes in crime outside of sub-beat 41 explains over 49 percent of the variance in crime rates within sub-beat 41, and the F-statistic shows we can be more than 99 percent certain that all of the estimated coefficients are different from zero. However, the residual diagnostics reported in Table 6 show that the residuals are neither normally distributed nor homoscedastic.

In the Town's geographical center, the change in crime rates in sub-beat 11 was found to be related to the change in crime rates in five other sub-beats. Two of these relationships, with sub-beats 12 and 13 to the south and southwest, were negative. For example, a 10 percent increase in crime in sub-beat 12 results, on average, in a 5.49 percent decrease in crime in sub-beat 11. Because these sub-beats are adjacent, it can be theorized that this negative relationship is due to criminals in the area moving between sub-beats and not usually targeting both sub-beats with greater frequency simultaneously. On the other hand, sub-beats 21, 40, and 41 each shows a statistically significant positive relationship in crime rates in sub-beat 11. Of these, sub-beat 41, to the southeast, displays the greatest magnitude. A 10 percent increase in crime in sub-beat 41 is predicted to correspond with an 8.07 percent increase in crime in sub-beat 11. The estimated linear association between crime rates in sub-beat 11 and other sub-beat explains one-third of the variance in crime rates in sub-beat 11. As before, the reported F-statistic indicates we can be 99 percent certain that all of the estimated coefficients are

different from zero. The residual diagnostic tests indicate that the residuals calculated from this regression are neither normally distributed nor homoscedastic.

Finally, in sub-beat 32 located in southwest Normal, crime rates are significantly associated in a linear form with four other sub-beats. The only negative coefficient indicates that a 10 percent increase in crime in sub-beat 31 leads to a mere 0.36 percent decrease in crime in sub-beat 32. Positive relationships were determined to exist between crime rates in sub-beat 32 and sub-beats 23, 33, and 41. Crime in sub-beat 23, on the far-east side of town, has the largest impact in terms of magnitude on crime in sub-beat 32: a 10 percent increase in crime in sub-beat 23 leads to a 4.63 percent increase in crime in sub-beat 32. The R-squared value indicates that nearly 20 percent of the variance in crime rates in sub-beat 32 is explained by variance in crime rates in the rest of town. According to the F-statistic, we can be 90 percent certain that the coefficients are statistically different from zero; yet according to the residual diagnostic statistics calculated, the residuals are again not normally distributed nor homoscedastic.

#### IV. Conclusions

Between the years 2004 and 2012, we have identified statistically significant relationships in crime rates across thirteen different police sub-beats in the town of Normal. We employed Ordinary Least Squares regression, using seven years of monthly data reported by the Normal Police Department. These linear associations likely speak to some movement, or flow, of crime between sub-beats. Negative relationships are found in six sub-beats adjacent to a dependent sub-beat and only one sub-beat non-adjacent to its dependent sub-beat. This could be expected, as it is simple for criminals to move between nearby sub-beats in order to find one ideal place to commit a crime. 18 out of

24 possible statistically significant positive relationships existed in sub-beats that were not adjacent to the dependent sub-beat which suggests co-movement of crime rates in sub-beats that are relatively afar.

This mobility of crime, or crime flow, is similar to that established by Smith (1976) in Rochester, New York and by Elffers, et al. (2008) in The Hague and The Netherlands. Similarly using local crime data recorded by the police department, approximate geographical centers of areas and the distance between them, and a control for population density, we found that the ease of mobility between sub-beats affects the relationship in crime rates between those sub-beats. In relation to the existing literature on gravity models of crime, this study could be improved with a more robust data set including a wider variety of crimes and more precise measures of population.

This study yields a series of policy implications. Regarding sub-beat 41, our findings indicate that increased law enforcement efforts west of Constitution Trail in sub-beats 10, 11, 12, 13, and 32, if successful in reducing crime rates, are likely to lower crime rates in sub-beat 41 to the southeast as well. In the same way, effectively decreasing crime rates east of Constitution Trail in sub-beats 21, 40, and 41 is likely to reduce crime in sub-beat 11. Keeping in mind that crime rates in sub-beats 12 and 13 are negatively related to crime rates in sub-beat 11, it would be useful to increase enforcement across this region and not only in sub-beat 11. Doing so would hamper nearby opportunities of crime and reduce the ease of crime flow between these regions. Lastly, concerning sub-beat 32, the identified statistically significant positive relationships among crime rates are all established with sub-beats to the east. In this case, lowering crime in sub-beat 32 would reduce crime rates in sub-beats 33, 41, and 23. Lastly additional crime-suppressing efforts



in sub-beat 31, to the east, will reduce the estimated crime flow between these two sub-beats.

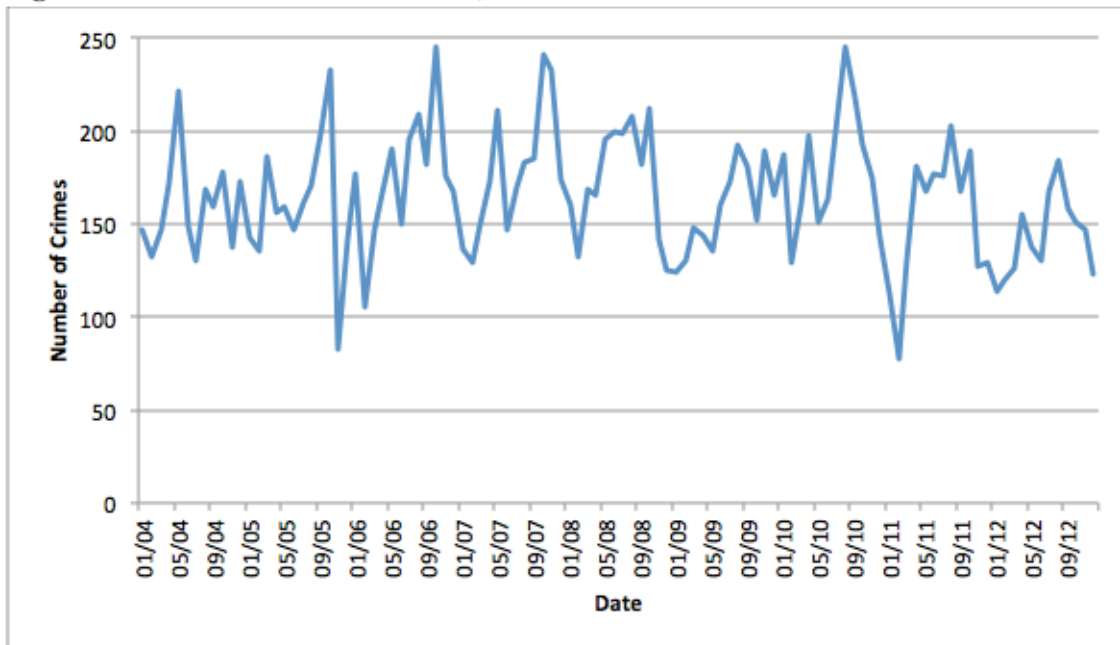
This study could be expanded in a number of ways. Because Normal is a twin city with Bloomington, Illinois, to the south, it is fair to assume that the mobility of crime does not stop at the edge of Town limits. Therefore, we will argue that crime flow between the two cities should be studied in depth and inform how Bloomington and Normal's police departments work together to reduce crime in the metropolitan area. Finally, crime recorded on Illinois State University's campus, in Normal, is not included in this data set, and though their reported numbers of the types of crimes used in this analysis are low, they would make the data set more complete. Also, a lengthier sample could uncover details of how crime rates change over time during economic expansions and contractions, at the time of significant local events (e.g. music concerts, football games), or throughout the seasons (e.g. summer vs. the rest of the year).

### References

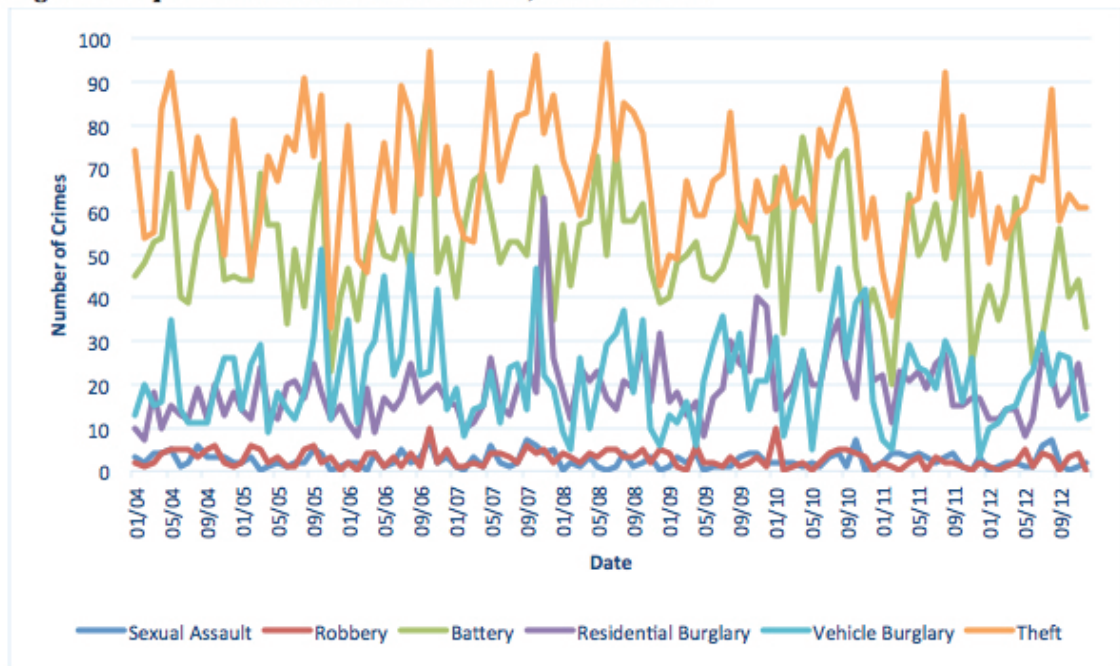
- Brantingham, P. (2011). Crime and place: rapidly evolving research methods in the 21st century. *Cityscape: A Journal of Policy Development and Research*, 13(3), 199-203.
- Elffers, H., Reynald, D., Averdijk, M., Bernasco, W., & Block, R. (2008). Modelling crime flow between neighbourhoods in terms of distance and intervening opportunities *Crime Prevention and Community Safety*, 10(2), 85-96.
- Kahane, L. H. (2013). Understanding the interstate export of crime guns: a gravity model approach. *Contemporary Economic Policy*, 31(3), 618-634.
- Normal Police Department. (2013). Crime Data [Data file]. Retrieved from <http://www.normal.org/index.aspx?nid=526>
- Smith, T.S. (1976). Inverse distance variations for the flow of crime in urban areas. *Social Forces*, 54(4), 802-815.
- Walker, J. & Unger, B. (2009). Measuring global money laundering: "the Walker gravity model." *Review of Law & Economics*, 5(2), 821-853.

## Appendix

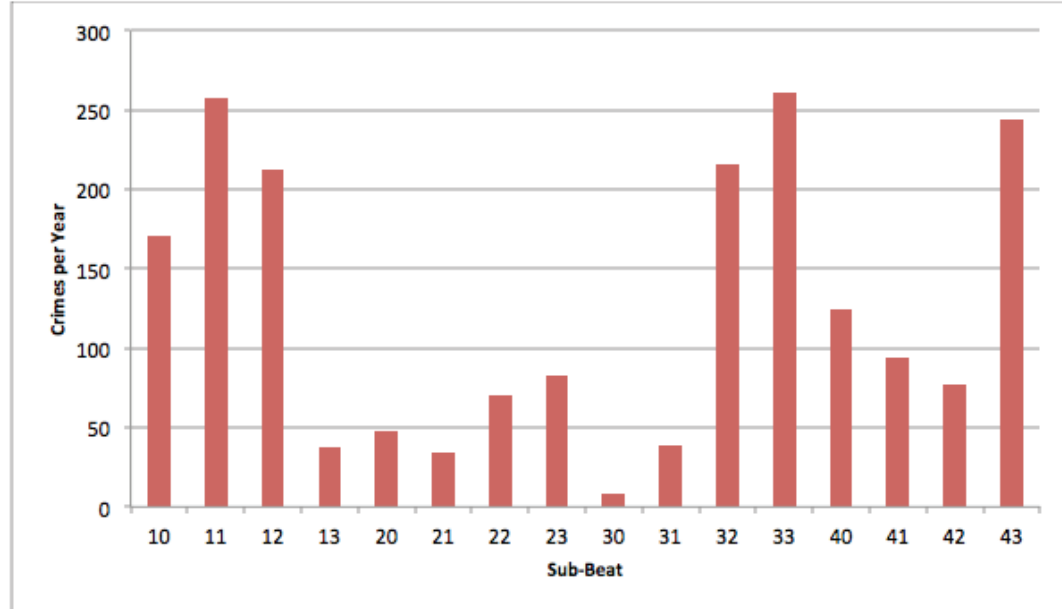
**Figure 1: Total Crime Series in Normal, IL: 2004-2012**



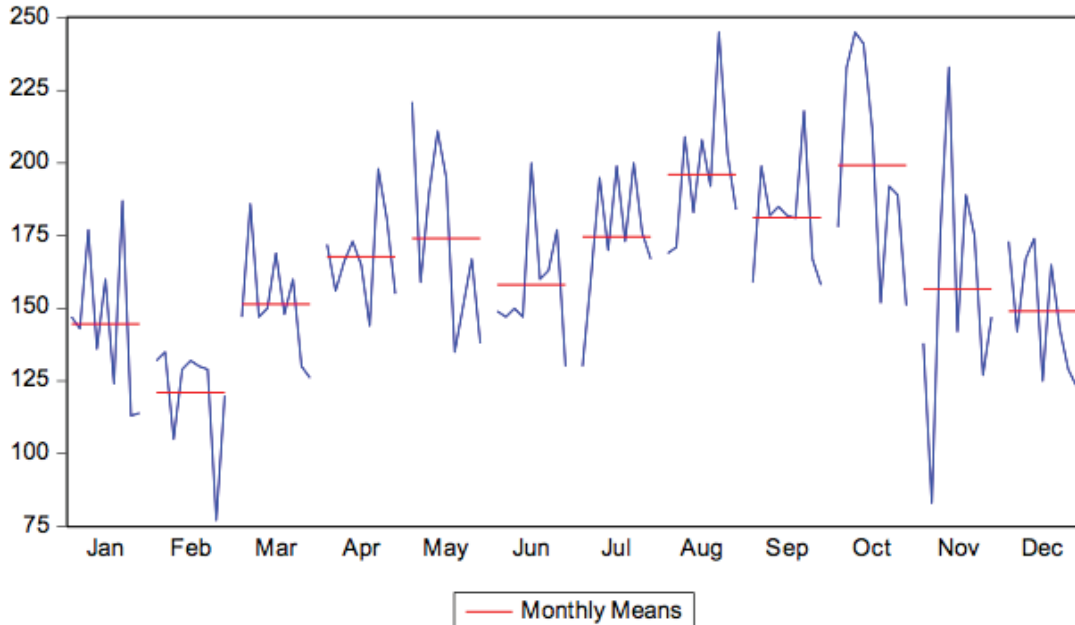
**Figure 2: Separate Crimes Series in Normal, IL: 2004-2012**



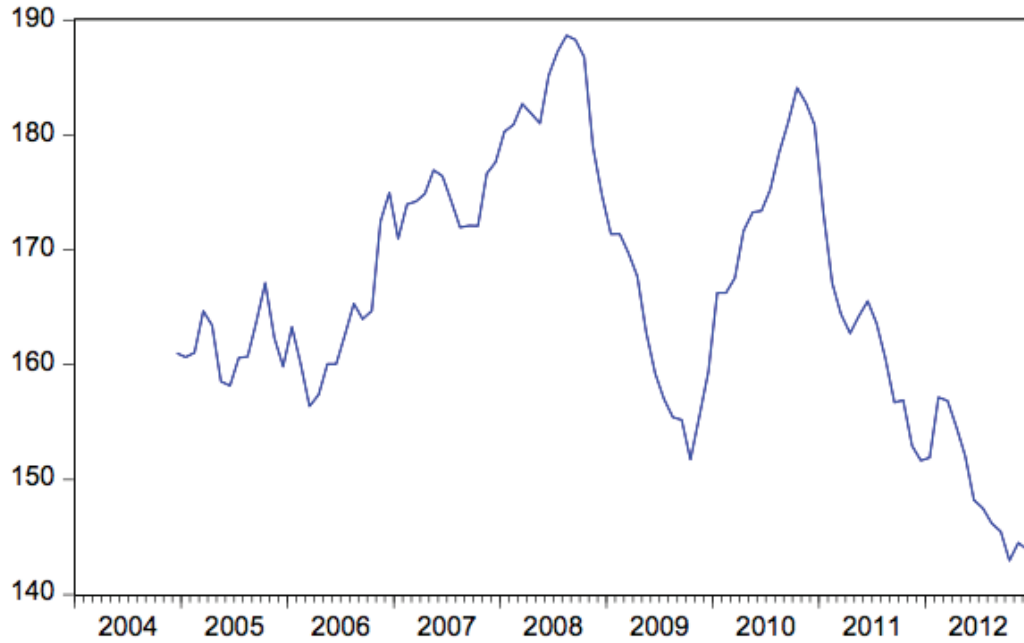
**Figure 3: Average Crimes per Year by Sub-Beat**



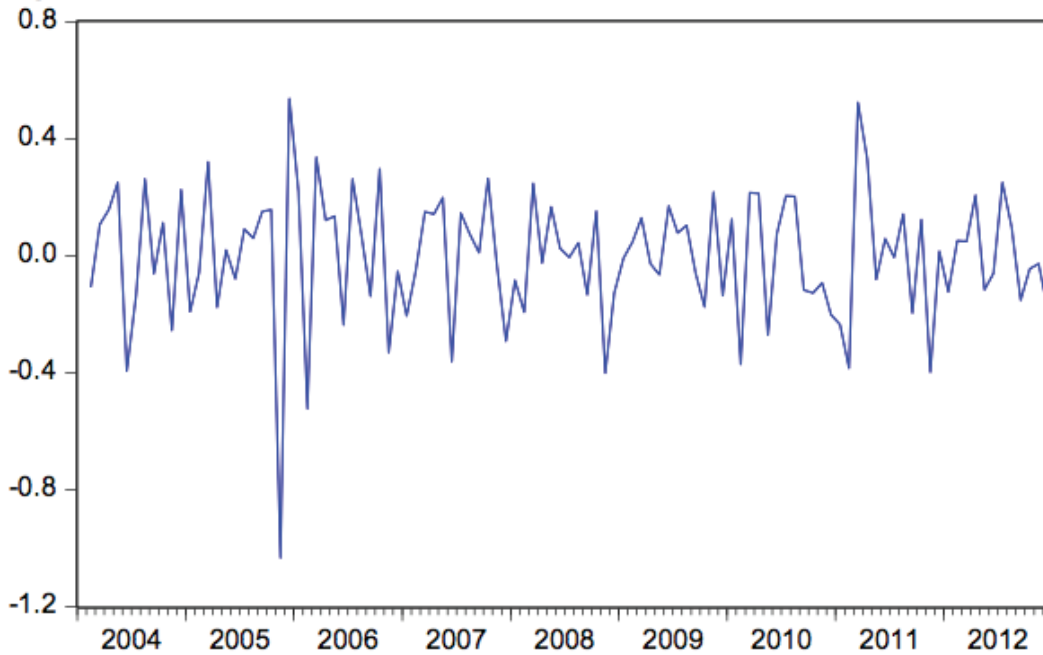
**Figure 4: Monthly Means of Crime in Normal, IL: 2004-2012**



**Figure 5: 12-Month Seasonally Adjusted Moving Average of Crime in Normal, IL: 2004-2012**



**Figure 6: Growth Rates in Crime in Normal, IL: 2004-2012**



**Table 1: ADF Unit Root Test (in levels)**

Variable in levels	t-Statistic
Total Crime	-6.5769
<i>Critical Values</i>	
1%	-4.0461
5%	-3.4524
10%	-3.1517

**Table 2: KPSS Stationary Test (in levels)**

Variable in levels	LM-Statistic
Total Crime	0.1517
<i>Critical Values</i>	
1%	0.2160
5%	0.1460
10%	0.1190

**Table 3: ADF Unit Root Test (in first-order differences of log values)**

Variable in f.o.d. of log levels	t-Statistic
Total Crime	-7.8503
<i>Critical Values</i>	
1%	-4.0565
5%	-3.4573
10%	-3.1546

**Table 4: KPSS Stationarity Test (in first-order differences of log values)**

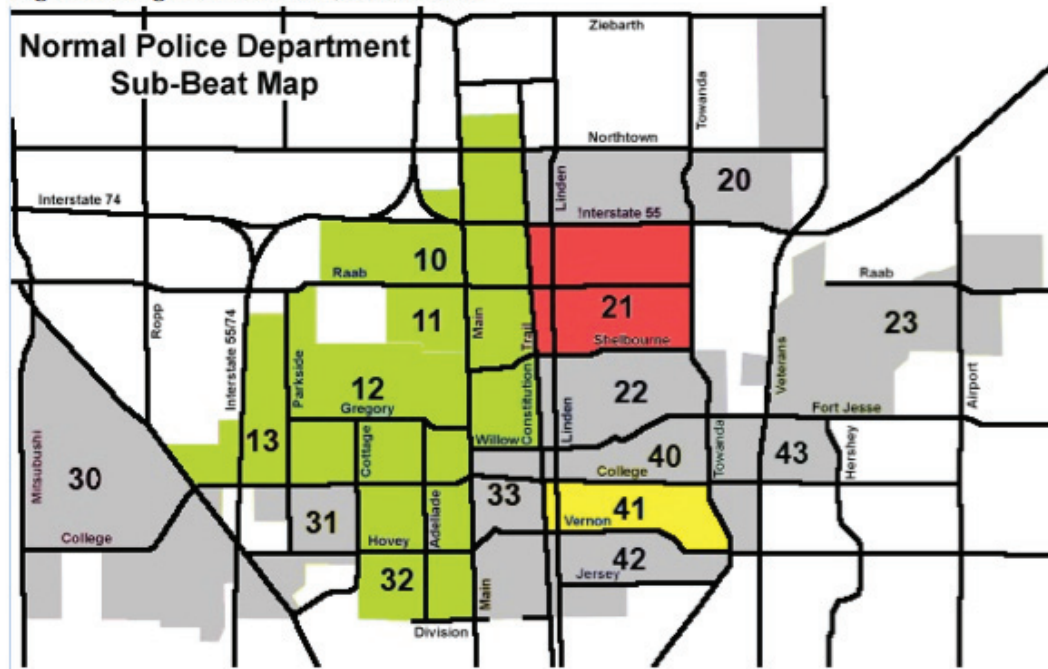
Variable in f.o.d. of log levels	LM-Statistic
Total Crime	-7.8503
<i>Critical Values</i>	
1%	-4.0565
5%	-3.4573
10%	-3.1546



**Table 5: Regression Results, Sub-Beat 41**

Dependent Variable: Sub-Beat 41	N=46
Constant	-0.0713 (-0.4632)
Sub-Beat 10	0.8870** (2.3390)
Sub-Beat 11	1.6876*** (3.7739)
Sub-Beat 12	2.2891*** (2.8280)
Sub-Beat 13	0.1671* (1.6899)
Sub-Beat 20	1.2405 (1.6436)
Sub-Beat 21	-0.5788* (-1.8836)
Sub-Beat 22	0.2244 (1.4455)
Sub-Beat 23	-0.1853 (-0.5919)
Sub-Beat 31	0.0269 (0.2277)
Sub-Beat 32	1.4878** (2.3254)
Sub-Beat 33	-0.5093 (-1.4116)
Sub-Beat 40	-0.0572 (-1.0606)
Sub-Beat 42	0.0719 (0.9405)
Sub-Beat 43	0.0193 (0.1723)
Adjusted R-Squared	0.4923
F-Statistic	4.1171***

Significance at the 1% (\*\*\*), 5% (\*\*), and 10% (\*) levels.  
T-statistics in parenthesis.

**Figure 7: Regression Results, Sub-Beat 41****Table 6: Jarque-Bera and Breusch-Pagan-Godfrey  
Residual Diagnostic Results, Sub-Beat 41**

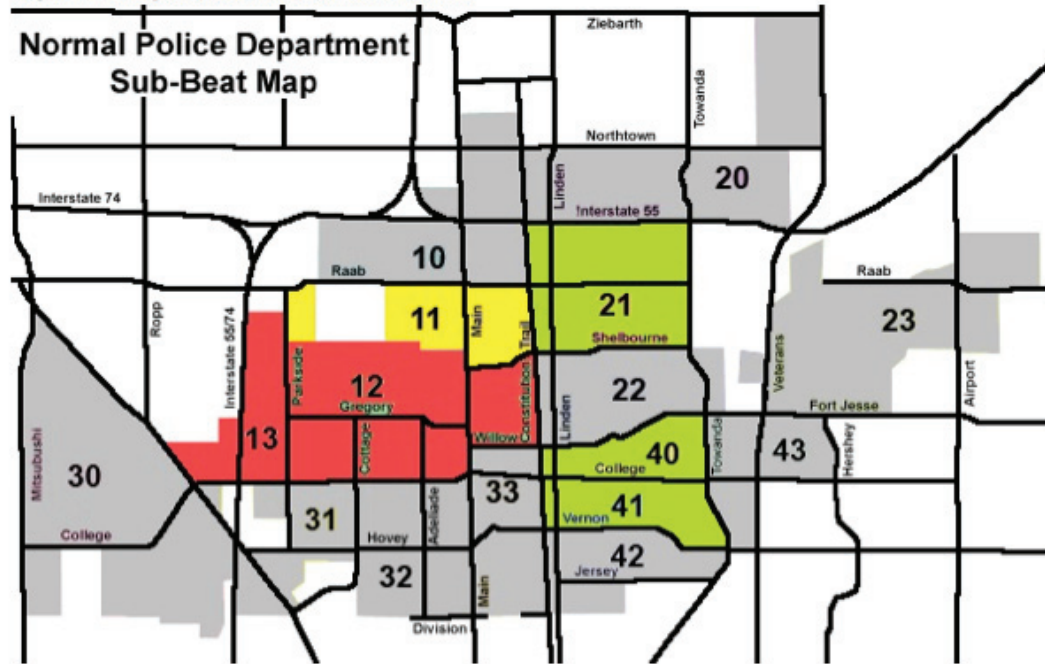
Residual Diagnostic Tests	Dependent Variable: Sub-Beat 41
Normality	1.4950 (p-value = 0.4736)
Heteroskedasticity	0.7859 (p-value = 0.6761)

**Table 7: Regression Results, Sub-Beat 11**

Dependent Variable: Sub-Beat 11	N=46
Constant	0.0793 (0.7496)
Sub-Beat 10	-0.0634 (-0.6876)
Sub-Beat 12	-0.5488*** (-2.9126)
Sub-Beat 13	-0.0918** (-2.4836)
Sub-Beat 20	-0.5481 (-1.1604)
Sub-Beat 21	0.3148* (1.7926)
Sub-Beat 22	-0.0826 (-0.4554)
Sub-Beat 23	0.2092 (0.6177)
Sub-Beat 31	0.0689 (1.1951)
Sub-Beat 32	-0.1130 (-0.2526)
Sub-Beat 33	-0.1840 (-0.5024)
Sub-Beat 40	0.3899** (2.3924)
Sub-Beat 41	0.8071*** (3.7739)
Sub-Beat 42	-0.2995 (-1.2274)
Sub-Beat 43	-0.0119 (-0.0550)
Adjusted R-Squared	0.3360
F-Statistic	2.6265***

Significance at the 1% (\*\*\*), 5% (\*\*), and 10% (\*) levels.  
T-statistics in parenthesis.

**Figure 8: Regression Results, Sub-Beat 11**



**Table 8: Jarque-Bera and Breusch-Pagan-Godfrey  
Residual Diagnostic Results, Sub-Beat 11**

Residual Diagnostic Tests	Dependent Variable: Sub-Beat 11
Normality	0.6216 (p-value = 0.7328)
Heteroskedasticity	0.9873 (p-value = 0.4880)

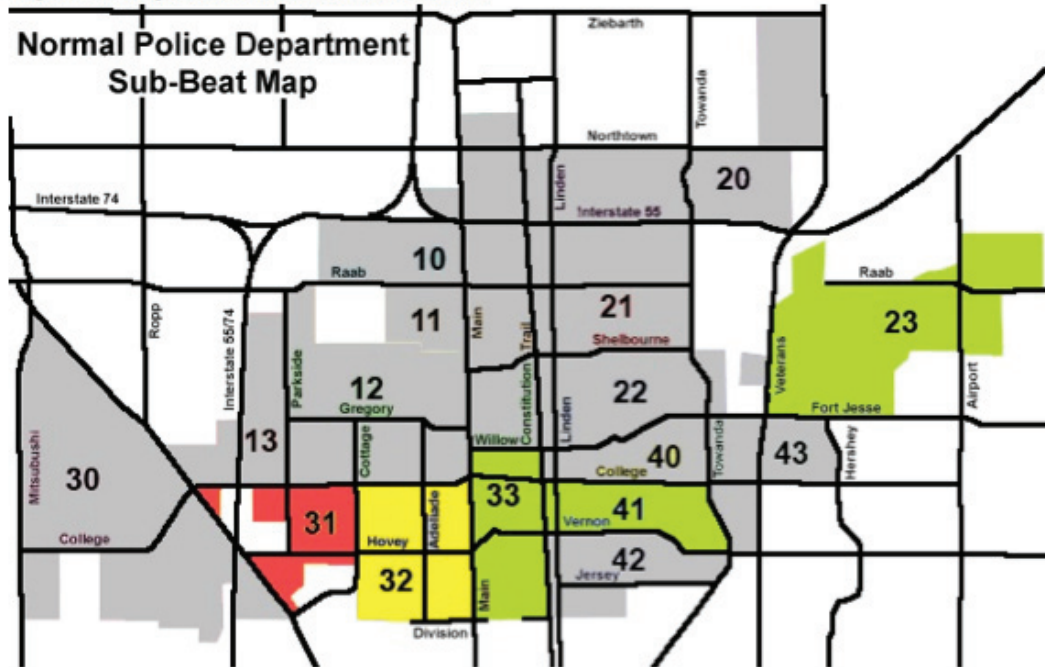
**Table 9: Regression Results, Sub-Beat 32**

Dependent Variable: Sub-Beat 32	N=46
Constant	0.0389 (0.5364)
Sub-Beat 10	-0.2847 (-1.4697)
Sub-Beat 11	-0.0525 (-0.2526)
Sub-Beat 12	0.1601 (0.6610)
Sub-Beat 13	-0.0105 (-0.4494)
Sub-Beat 20	-0.4068 (-0.7814)
Sub-Beat 22	-0.0611 (-0.3797)
Sub-Beat 23	0.4626* (1.7882)
Sub-Beat 31	-0.0357* (-1.9840)
Sub-Beat 33	0.2108** (2.0154)
Sub-Beat 40	-0.0325 (-0.2551)
Sub-Beat 41	0.3307** (2.3254)
Sub-Beat 42	0.0239 (0.1973)
Sub-Beat 43	-0.0353 (-0.2277)
Adjusted R-Squared	0.1966
F-Statistic	1.7894*

Significance at the 1% (\*\*\*), 5% (\*\*), and 10% (\*) levels.  
T-statistics in parenthesis.



**Figure 9: Regression Results, Sub-Beat 32**



**Table 10: Jarque-Bera and Breusch-Pagan-Godfrey  
Residual Diagnostic Results, Sub-Beat 32**

Residual Diagnostic Tests	Dependent Variable: Sub-Beat 32
Normality	2.7376 (p-value = 0.2544)
Heteroskedasticity	0.5309 (p-value = 0.8954)